Computer-Aided Determination of the Small-Signal Equivalent Network of a Bipolar Microwave Transistor

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Abstract—The actual technology for the production of transistors is now suitable for bipolar transistors in the 10-GHz range. In order to obtain amplifier circuits in this microwave range, the knowledge of the exact equivalent circuit is essential. A computer method for the determination of the equivalent network is given. The final application and accuracy are shown for a particular microwave transistor.

I. The Equivalent Circuit of an Intrinsic Transistor in Common Emitter Configuration and the Extrinsic Elements

On the basis of the experimental determination of the intrinsic elements and of the approximate equations by Te Winkel [1], we find the equivalent circuit of the intrinsic transistor (Fig. 1) which is only valid at small current density and at neglected recombination. The capacitances C_6 and C_7 are extrinsic elements. C_6 and C_7 take into consideration the RC line of the emitter-base junction in the backward direction [2], [10].

The resistance R_1 takes into account the resistance between the base connection and the internal base zone, which is doped slightly.

The transadmittance Y_{21e}' is complex. Pritchard [3] shows that Y_{21e}' may be approximated by a circle (Fig. 2). This circle approximation becomes worse at increasing frequency. Because we only want to find the equivalent network relative to a defined frequency range (in our case from 4-to-8 GHz), we approximate the transadmittance Y_{21e}' by the circle of Fig. 3, crossing the real axis at its zero point.

The current I' (see Fig. 1) may be expressed as follows:

$$I' = U_{b'e'} \cdot Y_{21e'} = \frac{U_{b'e'}}{R_3 + j\omega L_7} = -I''. \tag{1}$$

This equation can be represented by the additional network of Fig. 4. (It is possible to employ additional net-

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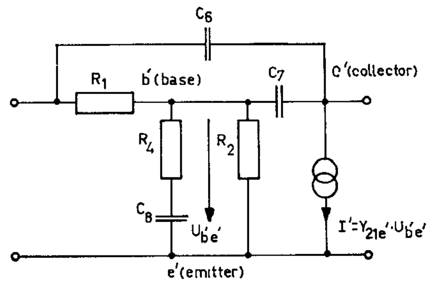

Fig. 1. Intrinsic transistor and extrinsic elements (C_6, C_7).

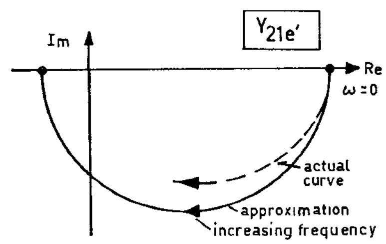

Fig. 2. Approximation of the transadmittance Y_{21e}'.

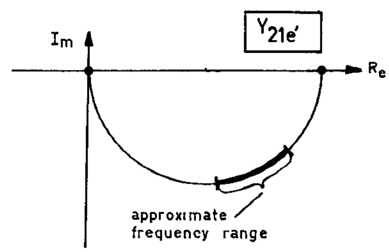

Fig. 3. Approximation of the transadmittance Y_{21e}' as employed in our computer program.

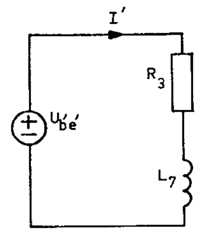

Fig. 4. Additional network for the transadmittance $y_{21e'}$.

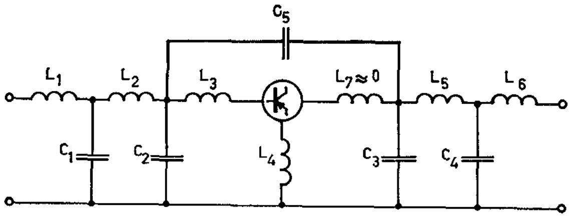

Fig. 5. Equivalent network of microwave transistor package. L_1, and L_6—lead inductances between the reference plane (scattering parameters) and the package. L_2, and L_5—lead inductances between the package edge and the gold wire connection. L_3, L_4, and L_7—inductances of the gold wires of the emitter, base, and collector lead-in wires to the transistor pill ($L_7 \approx 0$, because of the actual construction). C_1, C_2, C_3, and C_4—capacitances of the package. C_5—capacitance between input and output.

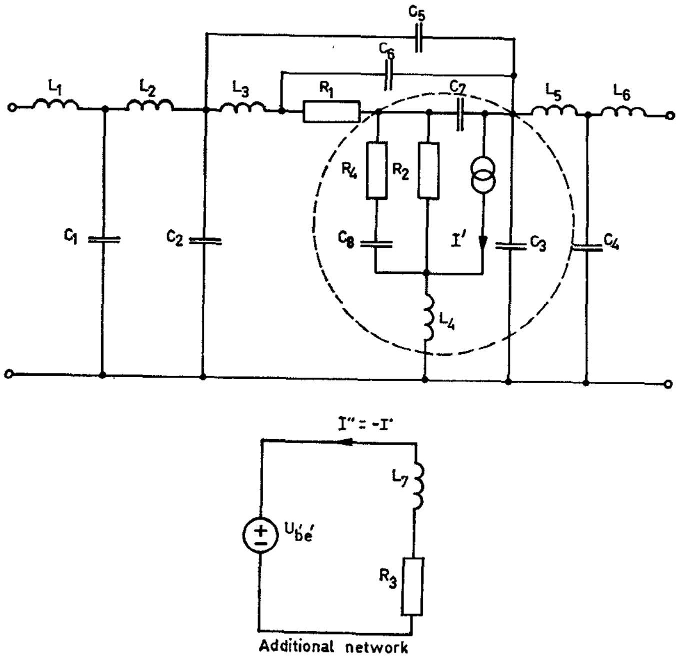

Fig. 6. Complete equivalent circuit and additional network for the complex transadmittance Y_{21e}'.

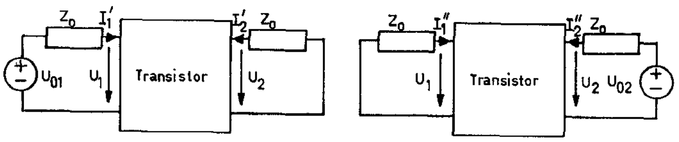

Fig. 7. Relationship between s parameters and the currents.

works for more complicated curves without impairing the validity of described method.)

The elements R_3 and L_7 are also found by optimization.

II. Equivalent Network for the Complete Transistor

For microwave transistors we have to consider the influence of the package as shown in Fig. 5. The ohmic parts of the leads (gold wires) may be neglected. Combining Fig. 5 with Figs. 1 and 4, we find the complete equivalent network of the transistor as shown in Fig. 6.

III. Computer-Aided Determination of the Equivalent Network Based on the Measured Scattering Parameters

A. Relation Between the Scattering Parameters and the Currents

As is well known [6], by definition of the scattering parameters the transistor is terminated at both ports by a pure resistance of value Z called the reference impedance. In our case, $Z = Z_0 = 50\ \Omega$. First a voltage source is applied at the input terminals and then at the output terminals (see Fig. 7).

If $U_{01} = U_{02}$ is equal to 1 V and Z_0 equal to 50 Ω, the following equations are valid.

1) *Input Excitation*:

$$I_1' = 0.01\ (1 - S_{11}) \tag{2}$$

$$I_2' = -\ 0.01 \cdot S_{21}. \tag{3}$$

2) *Output Excitation*:

$$I_1'' = -\ 0.01 \cdot S_{12} \tag{4}$$

$$I_2'' = 0.01 \cdot (1 - S_{22}). \tag{5}$$

B. General Procedure of Optimization

The equivalent network is found using a digital computer. The scattering parameters are measured at several frequency points of a specified frequency range and are compared with the calculated values. The elements of the equivalent circuit are varied until the error of the calculated parameters, as compared with the measured ones, is below a defined limit.

The equivalent circuit is found in two consecutive steps. First the transistor package alone (without pill) is measured [11], while the emitter, base, and collector connections are shorted using very short gold wires. These measured values within the frequency range under consideration are used for obtaining the first optimization of the scattering parameters of the package. For this the network of Fig. 5 is used. The optimization procedure is described in detail later. The initial values of the package elements, on which the said optimization is based, are found by estimation. The element values which we find by this method are needed only as initial values for the optimization of the complete network as, for example, the shorted gold wires are not as long as in the transistor, including the pill.

When this first optimization procedure is terminated, the complete network is taken into consideration (Fig. 6).

In the following sections only the optimization of the elements of the entire transistor is described because the package alone raises no special difficulties.

C. Determination of the Elements of the Transistor

Many optimization methods are based on the gradient of the error function. The solution of the present paper is based on the corresponding method of Fletcher and

Powell [4]. Because the said gradient has to be calculated frequently, it is very important for economical use of computer time that its determination be as simple as possible. Our gradient calculation is based on the excellent paper by Director and Rohrer [5]. However, their theory has to be extended, since the transadmittance Y_{21e}' is complex. The pill elements are only to be varied between defined limits.

D. Formation of the Error Function

For every frequency value the sum of the errors of the four scattering parameters has to be calculated. This is possible with two network calculations. First we calculate the input and output current with input and output excitation, respectively (see Section III-A). Unlike [5] we do not integrate over the frequency range. We calculate the sum of the squared errors at each frequency value. The minimum number of the individual frequency values is $2n$ if n is the number of the variable elements.

Sum E of the squared errors

$$= \frac{1}{2}\sum_{k=1}^{k=k_1}[W_1'(j\omega_k)\,|(I_1'(j\omega_k))_c - (I_1'(j\omega_k))_m|^2 + W_2'(j\omega_k)\,|(I_2'(j\omega_k))_c - (I_2'(j\omega_k))_m|^2 + W_1''(j\omega_k)\,|(I_1''(j\omega_k))_c - (I_1''(j\omega_k))_m|^2 + W_2''(j\omega_k)\,|(I_2''(j\omega_k))_c - (I_2''(j\omega_k))_m|^2] \tag{6}$$

where

k_1	number of frequency values $\geqq 2n$;
k	individual frequency value;
m	measured value;
c	calculated value;
$W_1'(j\omega_k)$, $W_2'(j\omega_k)$, $W_1''(j\omega_k)$, $W_2''(j\omega_k)$	weight functions;
$I_1'(j\omega_k)$, $I_2'(j\omega_k)$	input and output current with input excitation;
$I_1''(j\omega_k)$, $I_2''(j\omega_k)$	input and output current with output excitation.

Now, a further error sum function E_L is added to E. This second function increases rapidly if the variation of the network elements comes near its upper or lower limit, respectively.

Error sum E_L

$$= \sum_{i=1}^{i=n}\left\{\frac{(L_i)_{\text{lower}}}{X_i - (L_i)_{\text{lower}}} + \frac{(L_i)_{\text{upper}}}{(L_i)_{\text{upper}} - X_i}\right\} \tag{7}$$

where

X_i normalized network element (see III-E);
n number of variable network elements;
$(L_i)_{\text{lower}}$ ith lower limit;
$(L_i)_{\text{upper}}$ ith upper limit.

The total error sum E_T is

$$E_T = E + E_L \cdot W \tag{8}$$

where W is the weight function.

We could argue that this definition of E_L is not useful because the elements cannot attain their limits at the minimization process, as the corresponding values of E_L would be infinite. But we may choose the limit L_i beyond the exact physical limit. If this definition of E_L is used, it is very essential that the elements in the optimization subroutine program are only varied between the chosen limits L_i.

TABLE I

Element Type	Branch Relation	Branch Relation in Adjoint	Sensitivity (component of $\mathcal{G}$)	Component of Δp
Resistance	$V_R = R \cdot I_R$	$\psi_R = R \cdot \Phi_R$	$-I_R \cdot \Phi_R$	ΔR
Inductance	$V_L = j\omega L \cdot I_L$	$\psi_L = j\omega L \cdot \Phi_L$	$-j\omega I_L \cdot \Phi_L$	ΔL
Capacitance	$I_C = j\omega C \cdot V_c$	$\Phi_c = j\omega C \cdot \psi_C$	$j\omega V_C \cdot \psi_C$	ΔC
Current-controlled voltage source (see Fig. 11)	$V_{IDV} = Z_m \cdot I_{ICV}$	$\psi_{ICV} = Z_m \cdot \Phi_{IDV}$	$-\Phi_{ICV} \cdot \Phi_{IDV}$	

Note: V and I are the voltage and current in the original network; ψ and Φ are the voltage and current in the adjoint network.

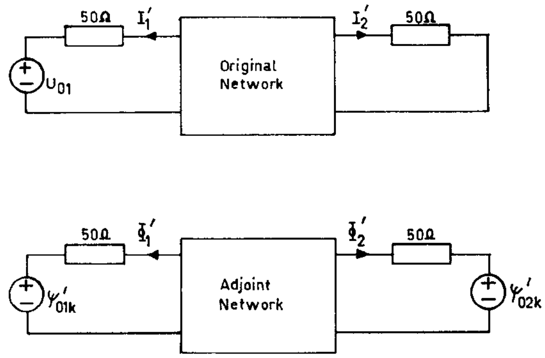

Fig. 8. Original and adjoint network with input excitation.

E. Calculation of the Gradient of the Error Sum Function E_T

1) *Gradient of the Error Sum Function E:* The calculation is based on the pairing of the originally specified network with its mutually adjoint network (see Fig. 13). This method makes use of a theorem of Tellegen [7]. With [5] and the definition of E one can derive the following equations:

$$\frac{\partial E}{\partial p} = \sum_{k=1}^{k=n}\left\{\text{Re}\left[W_1'(j\omega_k)\{(I_1'^*(j\omega_k))_c - (I_1'^*(j\omega_k))_m\}\frac{\partial(I_1'(j\omega_k))_c}{\partial p} + W_2'(j\omega_k)\{(I_2'^*(j\omega_k))_c - (I_2'^*(j\omega_k))_m\}\frac{\partial(I_2'(j\omega_k))_c}{\partial p} + W_1''(j\omega_k)\{(I_1''^*(j\omega_k))_c - (I_1''^*(j\omega_k))_m\}\frac{\partial(I_1''(j\omega_k))_c}{\partial p} + W_2''(j\omega_k)\{(I_2''^*(j\omega_k))_c - (I_2''^*(j\omega_k))_m\}\frac{\partial(I_2''(j\omega_k))_c}{\partial p}\right]\right\} \tag{9}$$

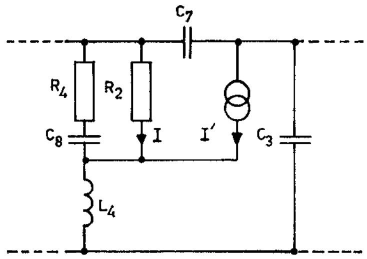

Fig. 9. Transformed network corresponding to Fig. 10.

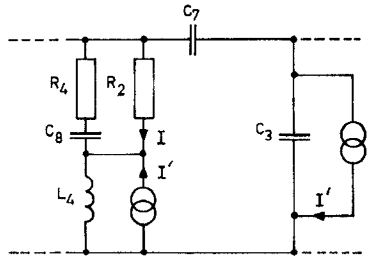

Fig. 10. Part of the original network of Fig. 6 as delimited by a broken line.

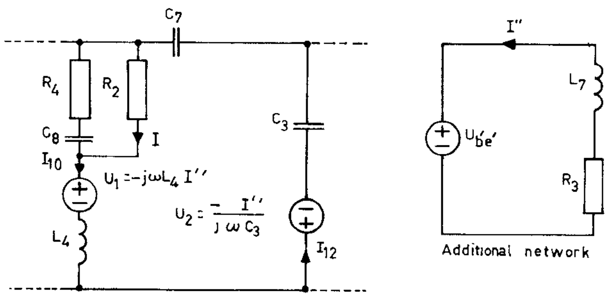

Fig. 11. Transformed network corresponding to Fig. 9.

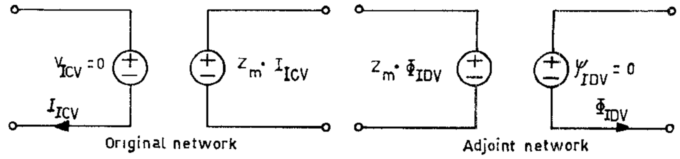

Fig. 12. Relation between original and adjoint network for a current-controlled voltage source.

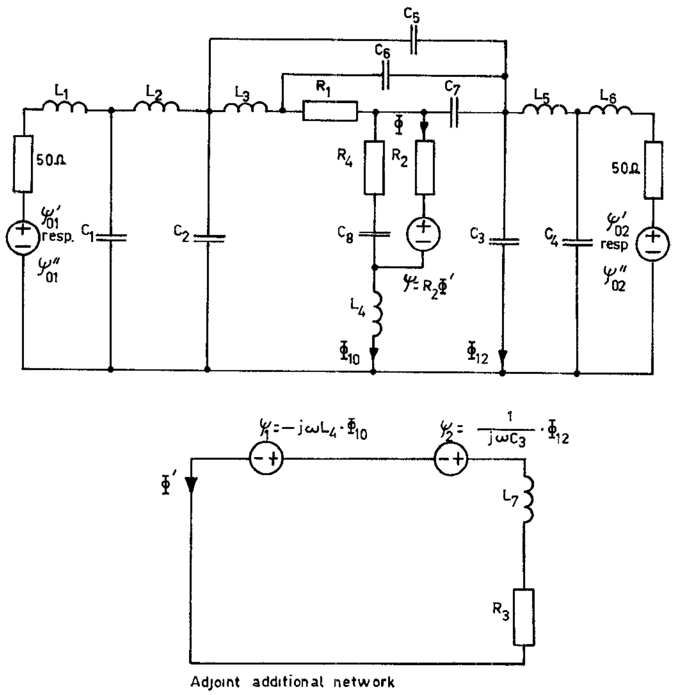

Fig. 13. Complete adjoint network and adjoint additional network.

where

p pth network element;

$$\frac{\partial E}{\partial p} = \sum_{k=1}^{k=n} \mathrm{Re}\,[\mathcal{G}'+\mathcal{G}''];\qquad(10)$$

$\mathcal{G}'$ first-order sensitivity with input excitation;
$\mathcal{G}''$ first-order sensitivity with output excitation;
Re real part.

In [5] the relations of Table I are derived:

Now we have to perform two network analyses, namely, with input and output excitation.

a) Input excitation (see Fig. 8; $U_{01}=1$ V, $U_{02}=0$ V.)*:* At every frequency value, the necessary branch voltages and currents of the original network are calculated. Because the input and output currents must have the measured values, we have to insert the following error voltage sources of Fig. 8.

Adjoint Network

Input Branch:

$$\psi_{01k}' = ((I_1'^*(j\omega_k))_m - (I_1'^*(j\omega_k))_c)\cdot W_1'(j\omega_k),$$
$$k = \text{frequency point.}\qquad(11)$$

Output Branch:

$$\psi_{02k}' = ((I_2'^*(j\omega_k))_m - (I_2'^*(j\omega_k))_c)\cdot W_2'(j\omega_k).\qquad(12)$$

Because the current directions of Fig. 8 are essential for this theory [5], we must change the sign of the relations of III-A; i.e., $I_2' = +0.01\cdot S_{21}$. The sensitivity $\mathcal{G}'$ is calculated according to Table I.

b) Output excitation: This procedure is completely analogous to the previous one.

c) Determination of the adjoint network: The determination of the adjoint network of the part of the original network in Fig. 6 which is marked by an interrupted line is not very simple because of the use of an additional network for the transadmittance Y_{21e}'. The adjoint network of the other part is equal to the original network. Fig. 10 is equivalent to Fig. 9, its current source having been transplaced. With a further source transformation we obtain Fig. 11.

We may consider $U_{b'e'}$ as a current-controlled (by I) voltage source and U_1, U_2 as current-controlled (by I') voltage sources. I' depends on I:

$$I' = U_{b'e'}\cdot Y_{21e}' = R_2\cdot I\cdot Y_{21e}'.\qquad(13)$$

According to [5] the current-controlled voltage sources are treated as shown in Fig. 12.

In the adjoint network we must insert the voltage

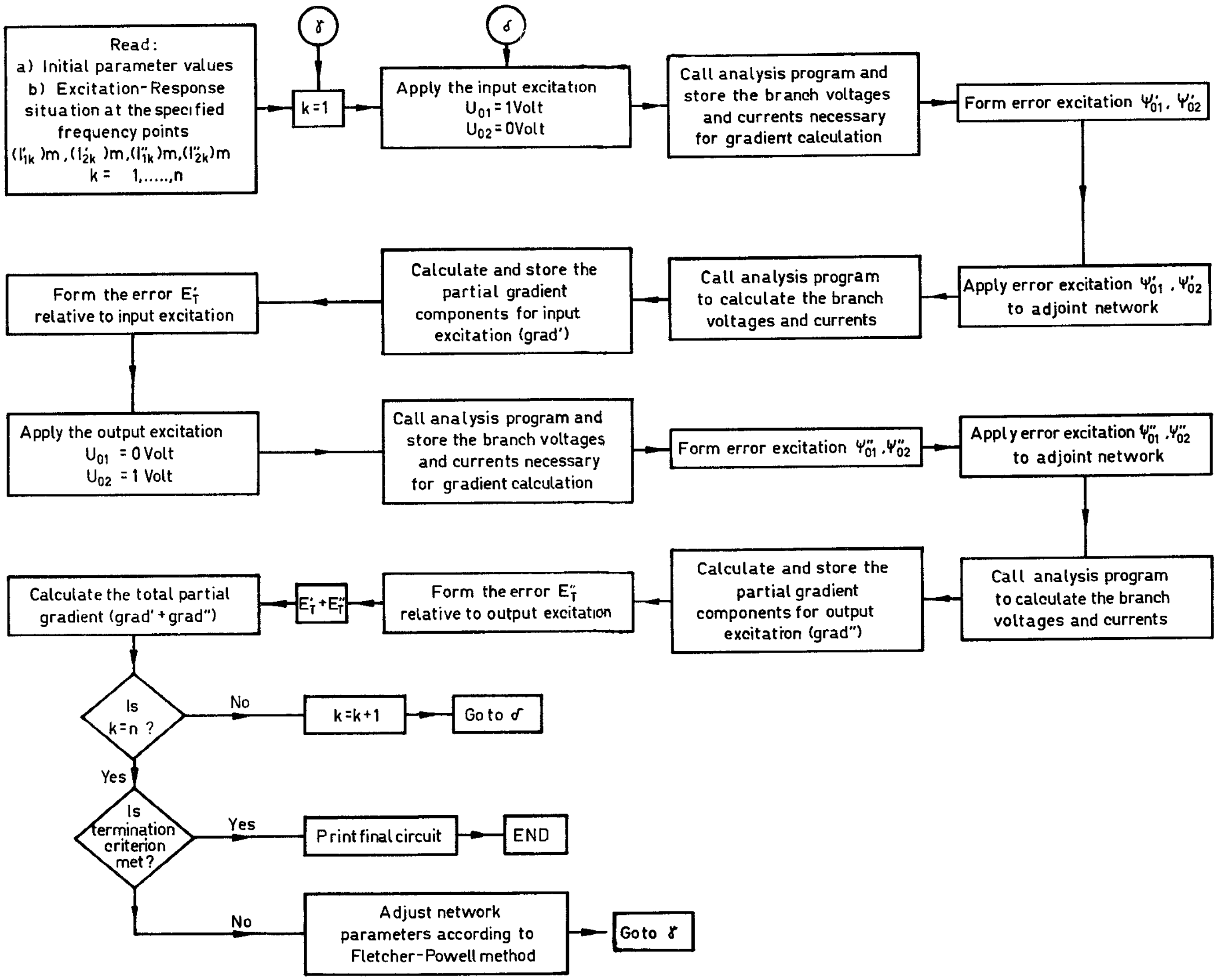

Fig. 14. Flow diagrams for computer optimization.

source (ψ) into the controlling branch. The voltage source of the controlled branch must be zero. Thus we obtain the adjoint network of Fig. 13.

Except for the controlling branch elements, the partial gradient components are calculated according to Table I. In the case of R_1 the gradient component becomes

$$\frac{\partial E}{\partial R_1} = \text{Re}\,\{\mathcal{G}' + \mathcal{G}''\}$$
$$= \text{Re}\,\{-(I_{R1}\cdot\Phi_{R1})' - (I_{R1}\cdot\Phi_{R1})''\}. \qquad (14)$$

For the sensitivity of the controlling branch elements (in our case R_2), the controlled current has to be considered. Thus the relationship of (14) has to be changed as follows:

$$\frac{\partial E}{\partial R_2} = \text{Re}\,\{\mathcal{G}' + \mathcal{G}''\}$$
$$= \text{Re}\,\{-(I_{R2}[\Phi_{R2} + \Phi_{R2}'])' - (I_{R2}[\Phi_{R2} + \Phi_{R2}'])''\}. \qquad (15)$$

The numerical determination of the gradient by contrast would require $2n$ calculations (n being the number of variable elements). Thus this method leads to an important time saving. The relationships of this chapter have been verified numerically.

2) Gradient of the Error Function E_L: This gradient can be determined by a simple analytical calculation.

3) Normalization of the Gradient and of the Network Elements: For the proper optimization subroutine program it is essential that the network elements be normalized to unity (initial value X_{i0}) at the start. Hence we need the normalized gradient components.

The total normalized gradient of the error function is

$$\frac{\partial E_T}{\partial X_i} = \frac{\partial E}{\partial X_i} + \frac{\partial E_L}{\partial X_i}$$

$$\frac{\partial E}{\partial X_i} = p'\cdot\frac{\partial E}{\partial p}$$

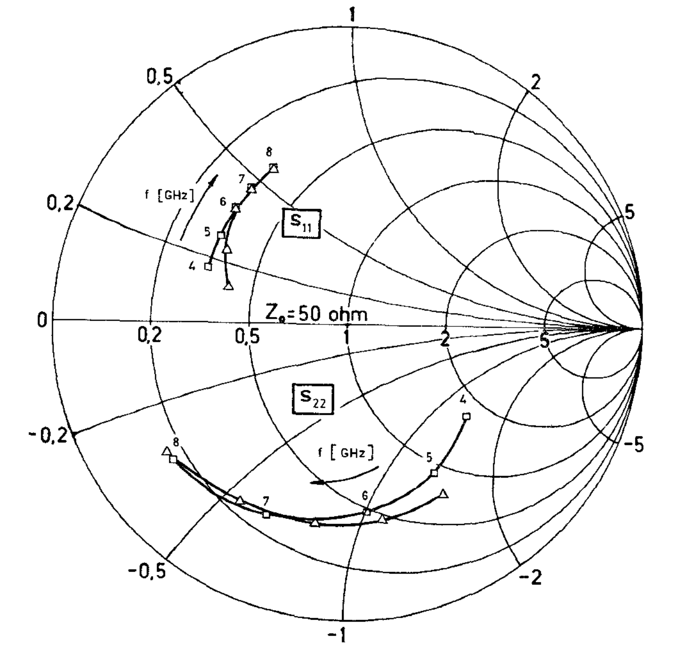

Fig. 15. Scattering parameters s_{11} and s_{22} (transistor Avantek AT-101A: U_{CE}, 10V; I_C, 3 mA). △ measured; □ calculated.

where

p' initial value of the network element;

p actual value of the network element.

This computer program has been applied to a microwave transistor in the frequency range from 4 to 8 GHz. Without the described gradient calculation the determination of the equivalent network by computer optimization would have been much more laborious (see [8] and [9]).

IV. Computer Programs

The flow diagrams of the computer programs used in carrying out the above procedure have been summarized in Fig. 14. The computer used for these programs is Control Data Type 6500, which is one of the computers of the Computer Center of the Swiss Federal Institute of Technology of Zurich.

V. Some Actual Results Obtained by Application of the Presented Method

As an example, the computed values of a microwave transistor (Type Avantek AT-101 A) at the following bias conditions are given:

Reverse Collector—Base Voltage 10 V
Collector Current 3 mA

Inductances (nH)	Capacitances (pF)	Resistors (Ω)
$L_1=0.21$	$C_1=0.085$	$R_1=$ 14.8
$L_2=0.20$	$C_2=0.011$	$R_2=279.8$
$L_3=0.16$	$C_3=0.001$	$R_3=$ 7.7
$L_4=0.09$	$C_4=0.013$	$R_4=$ 0.3
$L_5=0.31$	$C_5=0.005$	
$L_6=0.36$	$C_6=0.152$	
$L_7=0.42$	$C_7=0.316$	
	$C_8=4.011$	

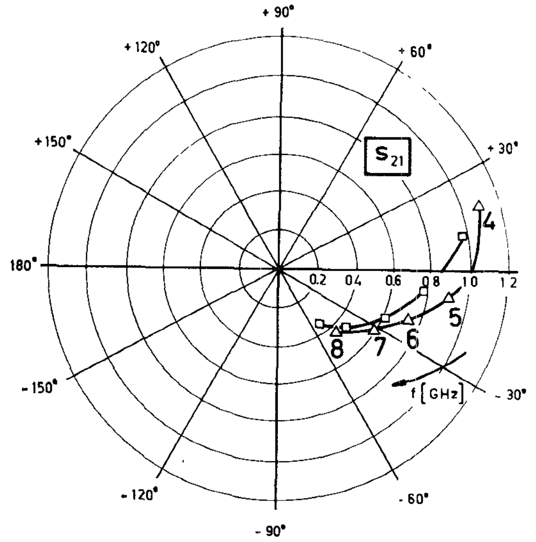

Fig. 16. Scattering parameter s_{21} (transistor AT-101A: U_{CE}, 10V; I_C, 3 mA). △ measured; □ calculated.

S_{12}

S_{12} MEASURED		S_{12} CALCULATED		f [GHz]
Re (s_{12})	Im(s_{12})	Re (s_{12})	Im(s_{12})	
0 136	0 032	0 145	0.023	4
0 146	0.005	0.158	0.006	5
0 145	-0 013	0 166	- 0 015	6
0146	-0 026	0 166	- 0 037	7
0.142	-0 035	0 162	-0 050	8

Fig. 17. Scattering parameter s_{12} (transistor AT-101A: U_{CE}, 10V; I_C, 3 mA).

This final model is valid in the frequency range from 4 to 8 GHz.

As a comparison, the measured scattering parameters, and those calculated with the final computed model, are shown in Figs. 15–17.

Acknowledgment

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Cavity Perturbation Techniques for Measurement of the Microwave Conductivity and Dielectric Constant of a Bulk Semiconductor Material

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***Abstract*—Cavity perturbation techniques offer a very sensitive and highly versatile means for studying the complex microwave conductivity of a bulk material. A knowledge of the cavity coupling factor in the absence of perturbation, together with the change in the reflected power and the cavity resonance frequency shift, are adequate for the determination of the material properties. This eliminates the need to determine the *Q*-factor change with perturbation which may lead to appreciable error, especially in the presence of mismatch loss. The measurement accuracy can also be improved by a proper choice of the cavity coupling factor prior to the perturbation.**

I. Introduction

THE COMPLEX microwave conductivity of a semiconductor material has been measured using two different methods. In the first one a semiconductor slab completely fills a waveguide section and measurements are made to determine the complex reflection or transmission coefficients. This method has been reported by many authors [1]–[3] and has been reviewed recently by Datta and Nag [4]. The accuracy achieved with the reflection method is not very precise, especially with high-conductivity materials, since the VSWR to be measured is very high (nearly 20 dB) and the phase-angle is very small. On the other hand, when this method is used in a transmission mode, the accuracy is degraded further due to available commercial standards for attenuators and phase shifters. The fact that the sample should completely fill the transverse cross section of the waveguide poses two serious problems. First, in many cases it is very hard to get large enough samples to fill the waveguide cross section; this problem becomes more serious when the sample is a single crystal. Second, there is always a small air gap between the sample and waveguide walls, even with tight fitting, and this effect was shown to give erroneous results [5]. Recently, Holm [6] suggested a mode transducer to overcome this "gap effect." Holm's idea takes advantage of the fact that the contact problem is important when the electric field is normal to the sample's surface, and by converting the TE_{10} mode of the rectangular guide into a TE_{10} mode of a circular guide the electric field is tangential to the surface of the sample at the boundary. This scheme may be difficult to realize with a brittle material like InSb.

The second method for measuring the microwave conductivity is by using cavity perturbation techniques [7]–[10]. The material parameters are measured by determining the *Q*-factor change and the resonant frequency shift of a resonant cavity by inserting the sample in the region of maximum electric field. The strong interaction between the fields in the cavity and the sample makes this method very suitable for the measurement of small changes in the material properties as a result of external perturbations. If the sample is placed under the central post of a reentrant cavity with a high *Q*-factor, and the size of the sample is chosen such that the energy stored within the sample is much smaller than the energy stored in the cavity, it can be shown that the change in the material parameters as a result of perturbing the sample can be related to the cavity

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